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ABSTRACT

Polya claims that true problem solving is accompanied by the cognitive activities of mobilization, organization, isolation and combination, and by the evaluations of relevancy, proximity, and quality. Evaluations occur as a result of monitoring cognitive activities. According to Polya, these particular cognitive activities are a necessary part of true mathematical problem solving. Furthermore, Polya claims emotion arises from these evaluations. The selection of problems which sample a broad range of mathematical areas and which evoke Polya's cognitive activities is discussed. The problem set presented here may be used as a single instrument, or in subsets of three parallel instruments. The problems are appropriate for students in grades 10 through 14. The problem set is designed for the investigation of mathematical problem solving. (Author/ASK)

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A Problem Set for the Investigation of Mathematical Problem Solving

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ABSTRACT

Polya claims that true problem solving is accompanied the by the cognitive activities of mobilization, organization, isolation and combination, and by the evaluations of relevancy, proximity, and quality. Evaluations occur as a result of monitoring cognitive activities. According to Polya, these particular cognitive activities are a necessary part of true mathematical problem solving. Furthermore, Polya claims emotion arises from these evaluations. The selection of problems which sample a broad range of mathematical areas and which evoke Polya's cognitive activities is discussed. The problem set presented here may be used as a single instrument, or in subsets of three parallel instruments. The problems are appropriate for students in grades 10 through 14. The problem set is designed for the investigation of mathematical problem solving.

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According to Polya (1981, 1985), there are certain cognitive activities that occur during true problem solving. These activities are mobilization, organization, isolation, and combination. During true problem solving, once a problem solver comes to an understanding of the statement of a problem, various pieces of information relevant to the problem must then be recalled from memory. Polya calls the process of extracting relevant elements such as formerly solved problems, theorems, and definitions, *mobilization*. Unconnected facts are adapted to the problem and grouped together.

The process of constructing connections between recalled information and the problem is called *organization*. The problem solver often must single out specific details of the problem one by one. By focusing on a single detail and isolating it from the rest of the problem, that detail can be better understood and its relevance and usefulness in solving the problem can be re-evaluated. This operation is called *isolation*.

By viewing the problem, or parts of the problem, from different standpoints and from different sides, the total conception of the problem can frequently be improved. Often, little progress can be made toward a solution without mentally varying one's point of view. By taking a different viewpoint, isolated details can be brought together into a new mental picture of the problem which may be a more harmonious combination of

details. A whole new "vue d'ensemble" or "Gestalt" may bring the problem solver closer to a solution. Polya calls grouping details together in a new way *combination*.

Polya considers mobilization and organization to be complementary activities in the way they work together. Similarly, isolation and combination are complementary activities. In proceeding toward the solution, the operations of mobilization, organization, isolation and combination may occur repeatedly and in any sequence. Progress may be slow and may take place in imperceptible steps. If the steps lead down a dead end path, the problem solver must repeat the mobilization and organization operations before further isolation and combination activities can occur.

Polya summarizes the activities that occur during problem solving by arranging his four operations on opposite corners of a square. The edges of the square represent cognitive activity sequences which are likely to occur repeatedly while solving a problem. These sequences include the operations of *recognizing* details, *remembering* relevant aspects of the problem, *supplementing* the problem with more information, and *combining* the information in new ways. Each sequence of activities should lead to an improved conception of the problem.

Polya contends that mathematical problem solving requires all of the mental resources that are present in his model. Thus, a

problem that does not require the activities of mobilization, organization, isolation, and combination, and their associated operations, is not a real problem but only an exercise. The distinction between a real problem and an algorithmic exercise is an important distinction in Polya's view.

Table 1 summarizes Polya's four major cognitive activities that are a necessary part of problem solving.

Table 1 Polya's four major cognitive activities which are a necessary part of problem solving

1. *Mobilization* occurs if: •familiar elements of the problem are recognized •relevant information is remembered
2. *Organization* occurs if: •details are added to supplement the problem •elements of a part of the problem are rearranged or grouped in a new way; a part of the problem which was in the foreground recedes into the background.
3. *Isolation* occurs if: •the problem is broken down into narrower parts; details are separated and made more distinct •a single detail is isolated, concentrated on, and studied
4. *Combination* occurs if: •all the details are combined in a new, more harmonious way •the whole problem is viewed in a new way.

The problems presented here are problems of the type described by Polya and involve all of his problem solving activities.

Three parallel problem sets (PS1, PS2, and PS3) were developed in two stages. An initial set of twenty-four problems was selected from over one-thousand problems of the type described by Polya. These twenty-four problems cover twenty-four different mathematical areas.

The problems were selected from the *Annual High School Contests of the Mathematical Association of America* (Salkind 1961), the *Annual Olympiad High School Prize Competitions in Mathematics* (American Academy of Actuaries 1970), the *Annual American High School Mathematics Examination* of the American Mathematics Competitions (Mientka 1990), and from a college level precalculus text (Swanson 1984).

The problems were selected at random from the one-thousand problems in order to get one problem in each of twenty-four different mathematical areas. However, problems were eliminated that were less representative of true problems in Polya's sense from the point of view of undergraduates who are required to solve them in one relatively short time period with no access to resources such as books, notes, and computers. For example, problems with the following characteristics were eliminated from the selection process: problems that depended only on remembering

a less well known mathematical rule or formula, problems that depended on familiarity with a more advanced mathematical method, problems that depended only on recognizing a single obscure aspect from the given information, and problems that depended only on drawing a single but not obvious auxiliary line.

The problems that were selected sample a wide range of mathematical areas from the general areas of arithmetic, algebra, geometry, and miscellaneous topics. Table 2 presents a detailed description of the problems in terms of mathematical area, Bloom's levels of cognitive complexity, and difficulty as measured by the number of steps to solve (see below). The classifications are not unique because the problem content is varied and the possible solution sequences are diverse.

Table 2 Selected problems classified in terms of cognitive complexity, and difficulty measured by the number of steps to solve

Problem	Math Topic	Bloom's Taxonomy Level	Number of Steps to Solve
1	geometry: solid geometry	analysis	14
2	arithmetic: number theory	comprehension	7
3	geometry: angle measurement	analysis	9
4	algebra: units conversion	analysis	10
5	algebra: linear equations	application	6

Table 2 Selected problems classified in terms of cognitive complexity, and difficulty measured by the number of steps to solve

Problem	Math Topic	Bloom's Taxonomy Level	Number of Steps to Solve
6	algebra: fractions	application	9
7	geometry: polygons	analysis	14
8	algebra: systems of equations	comprehension	6
9	algebra: ratio	application	14
10	geometry: coordinate geometry	comprehension	10
11	geometry: area	analysis	12
12	algebra: inequality	comprehension	6
13	algebra: progression	analysis	6
14	miscellaneous topics: sets	analysis	11
15	arithmetic: mean	comprehension	6
16	geometry: Pythagorean theorem	application	11
17	arithmetic: money	analysis	9
18	algebra: fractions	application	6
19	algebra: indeterminant equations	application	10
20	geometry: circles	application	10
21	algebra: quadratic equations	comprehension	7
22	arithmetic: percent	application	10
23	algebra: graphs	analysis	12
24	geometry: proportion	analysis	12

The diversity of the twenty-four problems can also be seen in terms of their key features and contextual dimensions (Nasser 1993). Table 3 presents the problems rated according to their key

features and contextual dimensions. Eighteen problems are in verbal presentation mode and six are in symbolic presentation mode. All of the problems translate into symbolic mode and all are convergent (have unique solutions). One problem is familiar in the sense that it is a familiar numerical pattern that is seen on a regular basis. All other problems are unfamiliar quantitative situations. Fourteen problems are imageable in that the elements of the problem can be imagined as physical objects. Ten problems are unimageable. Six problems deal with discrete variables and eighteen deal with continuous variables.

Table 3 Problems rated according to their key features and contextual dimensions

Problem	Familiar	Unfamiliar	Imageable	Unimageable	Symbolic	Verbal	Discrete	Continuous
1		X	X			X		X
2		X		X		X	X	
3	X		X			X		X
4		X		X		X		X
5		X		X		X	X	
6		X		X	X			X
7		X	X			X		X
8		X		X	X			X
9		X	X			X		X
10		X	X		X			X
11		X	X			X		X
12		X		X	X			X

Table 3 Problems rated according to their key features and contextual dimensions

Problem	Familiar	Unfamiliar	Imageable	Unimageable	Symbolic	Verbal	Discrete	Continuous
13		X	X			X	X	
14		X	X			X	X	
15		X		X		X		X
16		X	X			X		X
17		X	X			X		X
18		X		X	X			X
19		X	X			X	X	
20		X	X			X		X
21		X		X		X	X	
22		X	X			X		X
23		X		X		X	X	
24		X	X			X		X

The difficulty of each problem was measured according to the number of steps in a straightforward solution sequence (see appendix). Because each step presents some chance for an interruption or release from interruption (see Mandler 1984), the more steps in the solution, the more difficult the problem may be considered to be. A more detailed ranking would include the difficulty of each step in the solution, but this detail would not greatly alter the overall ranking of this particular problem set.

Eight problems required six or seven steps to solve and were classified as low difficulty problems. Eight problems

required nine or ten steps and were classified as medium difficulty. Eight problems required eleven or more steps and were classified as high difficulty problems.

Three parallel problem sets were designed to contain two low difficulty problems, two medium difficulty problems, and two high difficulty problems. However, because the three 14-step problems (#1, #7, and #9) are in some sense outliers from the other five high difficulty problems, each was assigned to a problem set. The rest of the problems were chosen at random from the low, medium, and high subsets. Problems were assigned to problem sets with the restriction that certain details in the solutions would not be repeated within a problem set. For example, if 30-60-90 degree triangles were involved in one problem, other problems in that problem set would not involve that particular topic.

In order to vary the order of presentation of problem difficulty, Problem Subset 1 (PS1) presents the problems in low-medium-high order. PS2 presents the problems in high-medium-low order. And PS3 presents the problems in medium-low-high order (see appendix). Table 4 shows which problems are assigned to the three problem subsets along with their difficulty level and their order of presentation.

Table 4 Problem assignment to subsets

PS1	PS2	PS3
low difficulty #5,12	high difficulty #7,14	medium difficulty #4,19
medium difficulty #17,22	medium difficulty #3,6	low difficulty #8,15
high difficulty #1,23	low difficulty #2,13	high difficulty #9,11

The problems may be scored using the California State Department of Education Assessment Program (Meier 1992). This scoring rubric scores problems as follows:

Exemplary response; score = 6.
The response is complete and includes a clear and accurate explanation of the techniques used to solve the problem. It includes accurate diagrams (where appropriate), identifies important information, shows full understanding of ideas and mathematical processes used in the solution, and clearly communicates this knowledge.

Competent response; score = 5.
This response is fairly complete and includes a reasonably clear explanation of the ideas and processes used. Solid supporting arguments are presented, but some aspect may not be as clearly or completely explained as possible.

Satisfactory with minor flaws; score = 4.
The problem is completed satisfactorily, but explanation is lacking in clarity or supporting evidence. The underlying mathematical principles are generally understood, but the diagram or description is inappropriate or unclear.

Nearly satisfactory, but contains serious flaws; score = 3.
The response is incomplete. The problem is either incomplete or major portions have been omitted. Major computational errors may exist, or a misuse of formulas or terms may be present. The response generally does not show full understanding of the mathematical concepts involved.

Begins problem but fails to complete solution; score = 2.
The response is incomplete and shows little or no understanding of

the mathematical processes involved. Diagram or explanation is unclear.

Fails to begin effectively; score = 1.
The problem is not effectively represented. Parts of the problem may be copied, but no solution was attempted. Pertinent information was not identified.

No attempt at solution; score = 0.
No attempt at copying or solving the problem is made.

The Program recommends first sorting the responses into three groups; good (5 or 6 points), adequate (3 or 4 points), or inadequate (0, 1, or 2 points). Each of these groups is then sorted.

The problem set presented here may be used as a single instrument, or as parallel subsets with students in grades 10 through 14. The problem set is designed for the investigation of mathematical problem solving.

24 PROBLEMS

1. Two spheres with one inch radii are situated so that each passes through the center of the other. Find the length of the curve of their intersection.

2. The price of an item was marked down from $0.30. If a number of them were purchased for a total of $4.81, how many were purchased?

3. What is the measure of the angle formed by the hands of a clock at 3:20?

4. Sally pays $1.20 for gasoline. She gets 20 miles per gallon from her car. She can have her engine overhauled for $300, after which she can expect to get 30 miles per gallon and save $1.00 per hundred miles on oil. After how many miles will she recover the cost of overhauling?

5. A farmer divides a herd of cows among his four sons so that one son gets one-half the herd, a second son, one-forth, a third son, one-fifth, and the fourth son, 7 cows. How large is the herd?

6. The fraction $\frac{5x-11}{2x^2+x-6}$ was obtained by adding the two fractions $\frac{A}{x+2}$ and $\frac{B}{2x-3}$. Find the values of A and B.

7. The base of an isosceles triangle is of length 6, and the altitude is of height 4. A rectangle of height x is inscribed in the triangle with the base of the rectangle in the base of the triangle. Find the area of the rectangle.

8. Is there an (x,y) pair that satisfies both the equations $2x-3y=7$ and $4x-6y=20$?

9. A radiator which has a maximum capacity of 10 quarts is filled with a mixture of 40% antifreeze. How much must be replaced with pure (100%) antifreeze to increase the strength of the original mixture to 60 percent?

10. Do the lines $y=3x+2$, $y=-3x+2$, and $y=-2$ form an isosceles triangle?

11. The circumference of a circle is 100 inches. How large a square can be inscribed in the circle?

12. For what values of x is $6x+1$ greater than $7-4x$?

13. At a dance party, a group of boys and girls exchange dances as follows: one girl dances with 5 boys, a second girl dances with 6 boys, and so on, the last girl dancing with all the boys. What is the relationship between the number of girls and the number of boys?

14. Twenty-nine students enrolled in a math course. Twice as many students went to the first class as went to the second class. If thirteen students went to both classes, and everyone that enrolled went to at least one class, how many students went to only the second class?

15. The average (arithmetic mean) of a set of 50 numbers is 38. If two numbers, namely 45 and 55, are discarded, find the mean of the remaining set of numbers.

16. The sides of a right triangle have lengths x, $x+y$, and $x+2y$ where x and y are positive values. Find the relationship between x and y.

17. Kim is now 20 and plans to live to be 100. She feels she will be able to save $2000 a year for the number of years she continues to work. If she estimates she will need $3000 for each retirement year, when can she retire?

18. Simplify the expression $\dfrac{x^2-3x+2}{x^2-5x+6} \div \dfrac{x^2-5x+4}{x^2-7x+12}$.

19. Can a student in a state with no sales tax spend exactly $1000 to buy philosophy books at $25 each and math books at $26 each. How would this be done?

20. Two of the angles on a triangle are 60° and the side between them is 4 inches. find the area of the triangle.

21. Does the equation $x-\frac{7}{x-3}=3-\frac{7}{x-3}$ have any solutions?

22. If the edge of a cube is increased by 50%, find the percent increase in the surface area of the cube.

23. A husband left his entire estate to his wife, his daughter, his son, and the cook. His daughter and son got half the estate, sharing in the ratio of 4 to 3. His wife got twice as much as the son. If the cook received the bequest of $500, what was the value of the entire estate?

24. A straight, 8 foot wide path is paved across the middle of a circular area 16 feet in diameter. How many square feet of the circular area is left unpaved?

SOLUTIONS

1. $l = 2\pi r = 2\pi\frac{\sqrt{3}}{2} = \pi\sqrt{3}$.

2. xp=481=13*37. p=13. x=37.

3. After 20 min. the hour hand is a third of the way from 3 to 4. 2/3 of 5 min. $=2/3(30^{\circ})=20^{\circ}$.

4. before: $1.20/gal * 1/20gal/mile = $.06/mile. After: $1.20/gal * 1/30 gal/mile = $.04/mile. Savings = $.02/mile + $.01/mile. $.03/mile X=$300 => X=10,000miles.

5. n=n/2+n/4+n/5+7. n=140.

6. $\frac{A(2x-3)+B(x+2)}{(x+2)(2x-3)} = \frac{5x-11}{2x^2+x-6}$. For x=-2, B=-1, x=3/2, A=3.

7. Let y be the base of the rectangle. Then
$\frac{4-x}{y} = \frac{4}{6}, y = \frac{3}{2}(4-x), area = \frac{3}{2}x(4-x)$

8. Parallel lines, no solution.

9. .40(10) - .40x + 1.00x = .60(10) => x=10/3 quarts.

10. The lines intersect pairwise at A=(0,2), B=(4/3,-2), and C=(-4/3,-2). $|AB|=|AC|=4\sqrt{10}/3$ and $|BC|=8/3$ so the triangle is isosceles.

11. 100=πd. d=100/π. let s be the side of the square. $s=d/\sqrt{2}=100/\pi\sqrt{2}=50\sqrt{2}/\pi$

12. 6x+1>7-4x => x>3/5.

13. With 1 girl at the dance there would be 5 boys, 2 girls, 6 boys, ... g girls, g+4 boys. Thus b=g+4.

14. (x=# in 1st class(y=#in both) z=# in 2nd class). Then x=2z, x-y+z=29 => z=28 => 1 student went to only the second class.

15. ave = sum of numbers (S)/ number of numbers (50). Thus, S=50 * 28 =1900, x=(1900-45-55)/48 = 37.5

16. $x^2+(x+y)^2=(x+2y)^2$. => x2-2xy+y2=0 => x=-y, the degenerate solution and x=3y.

17. x=# of years to work. Then 2000x - 3000(80 - x)=0 => x=48 years.

18. $\frac{(x-2)(x-1)}{(x-3)(x-2)} \bullet \frac{(x-3)(x-4)}{(x-4)(x-1)} = 1$

19. 25p + 26m = 1000 => p = 40 - 26m/25. The only positive integer solution is m=25, p=14.

20. The height of the equilateral triangle is $2\sqrt{3}$, so $A=\frac{1}{2}\cdot 4\cdot 2\sqrt{3} = 4\sqrt{3}$.

21. $x^2-6x-9=(x-3)^2$ => x=3 but the equation is not defined at x=3. Therefore, there are no solutions.

22. $A_{before} = 6x^2$, $A_{after} = 6(1.5x)^2 = 13.5x^2$; $13.5x^2/6x^2=2.25=225\%$.

23. 4x+3x = 6x+500 => x=500 => estate = $7000.

24. Area of sector = $\pi 8^2 120/360$; Area of triangle = $16\sqrt{3}$.

Area of unpaved area=2($\pi\, 8^2 120/360 - 16\sqrt{3}$).

PROBLEM SET #1

5. A farmer divides a herd of cows among his four sons so that one son gets one-half the herd, a second son, one-forth, a third son, one-fifth, and the fourth son, 7 cows. How large is the herd?

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